

Autumn 2013

# AIMatters

Newsletter of the American Institute of Mathematics

## SEEING THROUGH THE BIG BANG

Sir Roger Penrose and the  
Search for Truth Beyond  
the Known Universe



**The Mathematics of Planet Earth**  
Initiative Unites Global Academic Community

**From East to West**  
An International Team Tackles an Old Problem

**Prime Partners Exist**  
Progress Toward the Twin Prime Problem



# Letter from the Director

## The Latest News from Palo Alto



Greetings from Palo Alto! Yes, sad to say, we have still not moved yet. Our building project in Morgan Hill has been held up yet again, this time because of a PG&E gas pipeline that runs across the property not far from the building site. It remains to work out a way to proceed that

will satisfy everyone involved. I hope to have something better to report next year.

On a more positive note, it has been a good year for AIM. Lots of workshops, lots of SQuaREs, lots of good mathematics being done. Plus, 12 new Math Teachers' Circles, another REUF workshop, and interesting web developments! We were also involved in two successful public lectures.

Further, this year AIM was pleased to welcome two new employees: Stephanie Moore, Executive Assistant, and Hana Silverstein, Special Projects Assistant.

Mathematically, the biggest news is the proof that there are infinitely many pairs of primes which differ from each other by less than 5000! This fantastic new result builds on the work of Goldston, Pintz, and Yıldırım, some of which happened at AIM and was the subject of an AIM workshop. The other big math news is the solution of the Kadison-Singer problem. AIM had a workshop and three SQuaRE meetings devoted to this problem, which helped inspire a flurry

of work on the decades-old problem that has led to the solution.

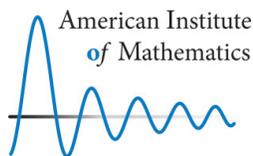
In March, AIM and MSRI collaborated in hosting Emily Shuckburgh of the British Antarctic Survey at the Palace of Fine Arts in San Francisco with her talk, "Climate Disruption: What Math and Science Have to Say." The talk was one of nine Simons' Foundation-sponsored lectures in the Mathematics of Planet Earth 2013 year. In May, AIM and Santa Clara University hosted Sir Roger Penrose of the University of Oxford for a delightful evening featuring his sold-out talk, "Seeing Through the Big Bang into Another World: The Power of Conformal Geometry."

Congratulations are in order to AIM's first Five-Year Fellow, Kannan Soundararajan, who recently received a Simons' Investigator Award, which is a 10-year grant to further his research program.

We are pleased that AIM continues to be the primary beneficiary of the highly successful Frys.com golf tournament. This year's event, which promises to be a good one, is scheduled for October 7 - 13 at CordeValle Golf Course in San Martin, Calif.

AIM is also pleased to announce that we are now the umbrella organization for the Julia Robinson Math Festivals. AIM looks forward to helping guide the program, which was initiated by Nancy Blachman and MSRI, in its next phases.

In this issue of AIMatters, you will also get to know two of our Scientific Board Members, get the latest news from our ongoing programs and get a glimpse into the latest mathematical work at AIM. Enjoy! ■



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# Table of Contents

## Letter from the Director

The Latest News from Palo Alto

2

## Seeing Through The Big Bang

Roger Penrose and the Search for Truth  
Beyond the Known Universe

4

## Mathematics of Planet Earth

Initiative Unites Academic Community

6

## Dispatches from Morgan Hill

Ten Years of Building the Future of Math

7

## Finding the Fun in Math

Julia Robinson Mathematics Festivals

8

## From East to West

An International Team  
Tackles an Old Problem

9

## A Drop into the Deep End

Young Mathematicians Find Their Way

10

## Solving the Unsolvable Together

A New Generation Tackles an Old Problem

10

## Supporting Research Mentors

REUF Program Continues to Thrive

12

## 100,000 Students and Counting

Math Teachers' Circle Network

13

## Meet the AIM Scientific Board

Hee Oh: "Mathematics is Beautiful"  
Alan Reid: A Double Life, Two Floors Apart

14

## Friends of the AIM Library

Special Thanks to Our Supporters

16

## Prime Partners Exist

Progress Toward the  
Twin Prime Problem

18

## From Our Collections

A Selection of Dissertations

20



# SEEING THROUGH THE BIG BANG

**Sir Roger Penrose  
Delights Audience  
with Tales of the  
World Beyond the  
Known Universe in  
Sold-Out Lecture  
Co-Sponsored by AIM**



# Is it possible that our Universe, the one that began about 14 billion years ago with what we call the “Big Bang,” is just one in an infinite succession of universes?

That was the question posed and analyzed by Sir Roger Penrose, one of the world’s most respected mathematical physicists, on May 20, 2013, in a public lecture at Santa Clara University, which sponsored the lecture along with AIM. The sold-out audience responded warmly to Penrose’s engaging style, and at the end of the hour he enthusiastically answered several questions of audience members, ranging from middle school students to professional physicists.

Penrose’s theory proposed that the entire history of the Universe that we know is just one era of an ongoing progression of universes, each of which begins with something like the “Big Bang,” whose evidence we see in the cosmic background radiation of our current universe. An especially intriguing aspect of the theoretical speculation is that the interaction of supermassive black holes in one universe leaves a trace of their existence that can be seen in the next universe—analogue to the way that a pebble thrown into a pond leaves evidence in the ripples of the water, even after it has disappeared beneath the surface. Toward the end of the talk, Penrose presented some observational evidence of these ripples in the microwave background radiation we see today, in effect “seeing through the Big Bang” to the previous universe, or “aeon,” as Penrose calls each of these universes.

Penrose has named his theory conformal cyclic cosmology—CCC, for short. The adjective “conformal” refers to the mathematics of space-time as a four-dimensional geometric object (or manifold) with a conformal geometric structure that describes the space-time coordinates and coordinate transformations that are allowed. Penrose explains CCC in detail in his most recent book, *Cycles of Time: An Extraordinary New View of the Universe*.

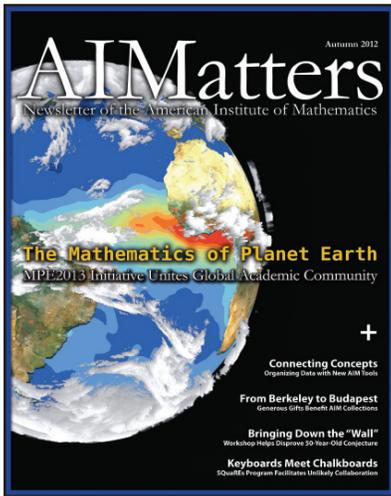
Penrose, who was knighted in 1994, is the Rouse Ball Professor of Mathematics (Emeritus) of the University of Oxford. During his long scientific career, spanning six decades, he has made fundamental contributions to our understanding of general relativity and cosmology, for which he was honored in 1988 with the Wolf Prize, shared with Stephen Hawking. That is just one of many awards, honors, and prizes bestowed on him. The earliest discovery for which he is known is aperiodic tilings of the planes, now called “Penrose tilings.” He also shares credit for the Moore-Penrose inverse of a matrix, and he developed the subject of twistor theory as part of a possible foundation for quantum gravity. He has written several books, among them *The Emperor’s New Mind*, a provocative best-seller proposing the theory that consciousness is a quantum mechanical phenomenon. ■

– Kent Morrison



Photo credits: this page and cover: Cosmic microwave background: ESA/NASA/WMAP; Penrose cartoon: Dismarup; Gunguiy via Wikimedia Commons

# The Mathematics of Planet Earth Initiative Unites Global Academic Community



The cover of last year's AIMatters newsletter featured the Mathematics of Planet Earth initiative. I am happy to report that now, as we ponder the activities halfway through MPE2013, the year has turned out to be tremendously energetic and successful.

The U.S. launch took place on January 10 at the Joint Mathematics Meetings in San Diego. It was held at the Math Institutes' reception, with over 300 in attendance enjoying food and champagne. The Joint Mathematics Meeting also featured many special sessions and two plenary lectures devoted to MPE2013.

The MPE2013 website at [mpe2013.org](http://mpe2013.org) highlights everything from a list of more than 60 workshops to special semesters, from curriculum materials and a speaker bureau to many other events and activities.

Two items that should be mentioned are the Simons Public Lecture Series and the daily blog. The lectures, which are taking place at nine locations around the world, each feature a leading expert who explains how the mathematical sciences play a significant role in understanding and solving some of Planet Earth's important problems. The second lecture in the series was jointly hosted by AIM and MSRI at the beautiful Palace of Fine Arts in San Francisco and featured Emily Shuckburgh of the British Antarctic Survey (pictured at right), who spoke on "Climate Disruption: What Math and Science Have to Say." A video of her talk and videos of three other lectures in this series are available on the MPE website.

Meanwhile, on the MPE2013 blog, over 150 posts have appeared, many of which make very interesting reading. AIM has been responsible for Thursday posts, and although we are not an institute that focuses on

applied mathematics, we have managed, with the help of many of our workshop and SquaRE organizers, to fill the Thursday blogs with a nice blend of mathematics and comments about general MPE issues.

AIM has also held three MPE-related workshops this year. The first, in January, was "Modeling Problems Related to Our Environment." The second, in February, was "Stochastics in Geophysical Fluid Dynamics," and the third, in June, was "Exponential Random Network Models." In the July 2 blog, one of the June workshop organizers, Persi Diaconis of Stanford University, illustrated the difficulties and successes of bringing people from different fields to work together. Diaconis writes, "The AIM workshop on exponential random network models was an experiment, bringing together people in applied social sciences, biologists, statisticians, and mathematicians who are interested in the emerging field of graph limit theory. All of us think about networks in some form or other, but the language, examples and aims are often very different... I don't think that the applied people had internalized some of the theoretical progress. Graph limit theory is full of infinite dimensional analysis and its main applications have been to extremal graph theory. After some of its potential applications were in believable focus, there was a lot of explaining and discussing." ■

– Estelle Basor



# Dispatches from Morgan Hill

## Ten Years of Building the Future of Mathematics

Now entering its tenth year, AIM's Morgan Hill Math program is going strong with a variety of programs that challenge fourth- through twelfth-grade students in mathematics. Highlights from this year include:

- A record total of 210 fourth, fifth and sixth graders took part in our fall Mathletics and MathCounts6 afterschool programs.
- Morgan Hill Math middle school students were winners at the regional MATHCOUNTS competition held in Salinas, Calif., on February 9. Three-year MATHCOUNTS veteran, Anthony Picchi, came in first place overall with the highest individual score, while the four-person team including Iris Indermitte, Tara Plummer, Jason Rhoads and Lauren Schirle from the Charter School of Morgan Hill clinched first place in the team category. Rounding out the list of Morgan Hill students who qualified to go to the Northern California State competition on March 23, was the inaugural team from Silicon Valley Flex Academy (Brandon Hoppa, Benjamin Snook, Eliesse Kwok and Sarah Kashani) and sixth grader Christopher Stawioski from the Charter School.
- Live Oak High School students James Gabbard, Katie Grant and Andrew Liu won first place in the Mathematics/Computers category at the Synopsys Silicon Valley Science and Technology Championship in March for their project entitled, "New Methods of Generating Sets of Intransitive Dice." This secured their position as Grand Prize alternates, and won them an expenses-paid trip to the California State Science Fair in Los Angeles.
- The Math Club at Ann Sobrato High School became an official Mu Alpha Theta chapter. The club ran a seminar at Science Alive! at Gavilan College in Gilroy, Calif. where they taught middle school students how to play the card game SET. Rachel McMillan, a math club leader and founding member of Mu Alpha Theta, was selected to attend the COSMOS summer program at UC Davis—studying mathematics, of course! ■

– Lori Mains



From top, Jennifer Kingman, an elementary school teacher and a coach in AIM's Morgan Hill Math program, poses with her fourth and fifth grade Mathletics class in front of a quilt they created. The design of each quilt square incorporates math concepts that the students studied, such as the pattern of a base 4 addition table and reflectional symmetry. Below, the Sobrato High School Math Club at Science Alive! at Gavilan College in Gilroy, Calif.

# Finding the Fun in Mathematics

## AIM Teams Up with Julia Robinson Math Festival



In July, AIM announced a new partnership with the Julia Robinson Mathematics Festival, which, since its first event at Google in 2007, has offered a nurturing environment that has inspired and engaged over 7,500 elementary, middle, and high school students to develop an interest and find fun in mathematics. Named after Julia Robinson, a math professor at the University of California, Berkeley, known for her solution of Hilbert's Tenth Problem, the Festival encourages collaboration, exploration and creative problem solving.

Nancy Blachman, a co-founder of the event together with MSRI, explains, "We founded the Festival to give children an opportunity to explore fascinating problems, free from the more usual competitive environment that scares off many talented students. At a Festival, there are numerous tables with mathematical problems or puzzles that require creativity and persistence to solve, and each table is run by one or two facilitators—many with mathematical backgrounds and teaching experience—to offer help and encouragement."

Festival director Joshua Zucker recalls an early Festival at which computer scientist Donald Knuth, volunteering as a facilitator, was approached by a sixth-grade girl. "Can you help me with this problem?" she asked. "I don't know," said Knuth. "Can you explain the question to me?" Zucker, who has worked with AIM on other projects, including Math Teachers' Circles, was the first to suggest the partnership to AIM director Brian Conrey.

"Developing future generations of mathematicians is an essential part of AIM's work," said Conrey. "We've been very impressed with how effectively the Julia Robinson Mathematics Festival has been engaging kids around the country and showing them a side of math very few of them see in school. We're happy to add this new dimension to our educational outreach."

"AIM's connection to teachers makes this a natural partnership," says Blachman. "We were really pleased at Brian's interest in the Festival and his ideas and resources for making it better. We're looking forward to being able to bring large and small festivals to many more kids in the next year and beyond." ■

– Mary Eisenhart



From top: Students discuss puzzles with a facilitator. Two students work side by side on the same problem, which is encouraged at the Festivals in order to make math more appealing to many students. A facilitator gives kudos to a student for a problem well solved.

Photos: Katelyn Weingart.

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# From East to West

## An International Team Tackles an Old Problem

During the August 2010 Workshop on Differential Structures on Finite Sets at AIM, fellow participant Andreea Nicoara started a discussion with me regarding some problems related to Denjoy-Carleman classes of infinitely differentiable functions. I had worked in this area from an analytic point of view, mostly at the beginning of my career in the 1970s. Nicoara was interested in algebro-geometric aspects of this area, which have attracted quite a lot of attention of people in real algebraic geometry in the last decades. Very soon it became clear that a number of old results, mostly by mathematicians from the former Soviet Union, could be quite relevant to the problems Nicoara was interested in. These results were not well known in the West.

In August 2011, at a Whitney Problems Workshop at the College of William and Mary, we continued this discussion, joined by Michail Bronshtein of the Federal Research Technological University in Kazan, Russia. Bronshtein and I have known each other since high school, and have previously collaborated on problems related to analytic aspects of Denjoy-Carleman classes, mostly inspired by questions arising in Partial Differential Equations. After quite fruitful discussions, it became clear that study of algebraic structure of Denjoy-Carleman quasi-analytic classes can be successfully approached from the analytic point of view.

We decided to apply to AIM's SQuaRE program and invited Francesca Acquistapace and Fabrizio Broglia of the University of Pisa, Italy, to join us. Acquistapace and Broglia are well-known experts in algebro-geometric aspects of quasi-analytic Denjoy-Carleman classes, and their insights were certain to be extremely helpful.

Before we first met in August 2012, we began with an attempt to mimic an approach successfully used by Bronshtein in the non-quasi-analytic case, trying to find a larger quasi-analytic DC class where any function from the initial class can be prepared. After a number of failed attempts, we realized that there is a serious obstacle to this approach: the absence of suitable extension results for these DC classes.

Finally, we were able to prove a very strong non-extension theorem for quasi-analytic classes, which showed that for any given quasi-analytic DC class on a half axis and any larger quasi-analytic class on the whole axis, one can find a function in the first class not extendible to a function of the second class. The construction is very explicit, which opens new possibilities for its application to other problems in the field.

As a consequence of this non-extension result, we were able to show that given any quasi-analytic DC class in two or more variables, and given any larger quasi-analytic DC class, one can find a function in the first class (satisfying the usual Weierstrass conditions) that cannot be prepared in the larger class.

The paper with our result was posted on the ArXiv in December of 2012 and it immediately attracted quite a lot of attention. Experts in the field started to try to find easier proofs of our results, but so far without much success. Only substantially weaker results (not taking into account the possibility of widening of the class) were obtained.

We followed up with a second meeting at AIM in June of 2013. We decided to use little known but powerful ideas of pseudoanalytic continuation (developed mostly by E. Dynkin in the 1970s). These ideas have led us to an understanding that there is a natural localization of a quasi-analytic DC class (it is an integral domain, after all!) which provides an excellent setting for Weierstrass division, and, therefore, for Weierstrass preparation. An important feature of this localization is that the main idea of quasi-analyticity (injectivity of the Taylor mapping) remains intact.

We are currently working on these ideas, and we already have many plans for our next meeting in August 2014.

Our work has confirmed the wonderful effectiveness of bringing together mathematicians with very different mathematical backgrounds to attack outstanding problems of common interest. And the SQuaREs program at the American Institute of Mathematics provides an effective vehicle for such collaboration. ■

– Nahum Zobin

# A Drop into the Deep End

## Young Mathematicians Find Their Way at AIM



Photo: Lisa Traynor.

*Sabloff, Henry, and Rutherford on a hike.*

In September 2008, Brad Henry of Siena College was a graduate student at Washington University. “I was in my sixth year,” he explained. “I was intending to finish in the spring, and was fairly far along in a project that I believed would bear fruit, enough so I could finish my doctorate.”

Henry’s project involved trying to understand connections between two approaches to Legendrian knots. One approach came from symplectic field theory and the other came from classical Morse theory, and there was evidence that these two approaches were related. However, there were some unresolved issues that were stalling his thesis.

That was when his advisor, Josh Sabloff of Haverford College, invited him to an AIM workshop on Legendrian knot theory. The first day of the workshop, Sabloff came to Henry with a request. “He said he’d like me to give a talk the next day on my research problem,” said Henry. “This is Josh’s style, to drop you in the deep end and see what happens.”

It was daunting to give a one-hour talk on a research project that he hadn’t completed. “There were a lot of experts in the field,” Henry said. “Many names I’d seen in papers but never seen in person. There weren’t many graduate students there.” But the audience was friendly, and the talk was conversational.

“I described the problem I was working on and the tools I was using,” Henry said. “At one point, Dan Rutherford of the University of Arkansas asked how I would account for a certain problem that arises—how did my work handle that issue? The issue he brought up, unbeknownst to me, was the issue that was keeping my project from being successful, and the tools I was using to address this issue were insufficient.”

A group of eight participants broke off for the afternoon sessions to tackle some of the issues Rutherford had brought up during Henry’s talk. “At the end

of the week, the powers that be, as I thought of the tenured people in our group, got together to discuss what my thesis project should become,” Henry said. “The idea was to adapt a new tool we’d been introduced to the previous day to try to solve something related to my original thesis problem.”

He had been hoping to finish in May. “I went into frantic work mode,” he said. “I finished in July.” Afterwards, he and Rutherford, who had also recently finished his doctorate, formed a collaboration. Their first paper was published in January, and their second is nearly complete.

Five of the eight workshop participants who broke off for the afternoon sessions also ended up forming a SQuaRE, or Structured Quartet Research Ensemble. SQuaREs are groups of four to six mathematicians who spend a week doing research at the AIM headquarters in Palo Alto, Calif., with the possibility of returning for a week at a time in subsequent years.

Henry’s SQuaRE—consisting of himself, Rutherford, Sabloff, and Lisa Traynor and Paul Melvin of Bryn Mawr College—is in its third year. They continue exploring connections between the two approaches to Legendrian knots. “We have a better understanding with each new day of what’s happening with these two approaches,” Henry said. “Without the 2008 workshop, I likely wouldn’t have finished my thesis. I wouldn’t have solved my problem, I wouldn’t have met Dan, at least not in that small setting, which I think made a big difference. I’ve never been to a workshop where I was more productive than here. The format is ideal, I think, for research mathematics.” ■

– Hana Silverstein

### CALL FOR PROPOSALS

Proposals are currently being sought for week-long workshops for up to 28 people and SQuaRE collaborations for 4-6 researchers to take place in 2014-2015 at AIM in Palo Alto, California.

Application deadline: November 1, 2013.

# Solving the Unsolvable Together

## A New Generation Tackles an Old Problem

Kevin Pilgrim of Indiana University organized a SQuaRE in July to work on a problem that has kept him and his colleagues occupied for the past decade: analyzing complex dynamics and selfsimilar groups. How did this SQuaRE come to be? The answer is part of a larger story of mathematical collaboration.

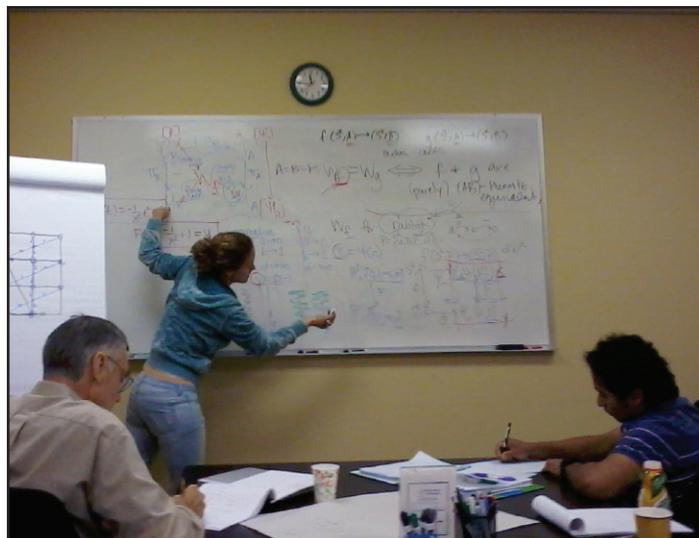
In the early 1980s, William Thurston characterized certain types of dynamical systems by combinatorial conditions. However, these conditions were hard to work with. In fact, a quite simple problem (which later became known as the “twisted rabbit problem”) remained unsolved for over a decade.

But in 2006, Laurent Bartholdi and Volodymyr Nekrashevych solved this problem by introducing a new technique using ideas from algebra and geometry. “Since that time,” explained Pilgrim, “there’s been great synergy between people interested in selfsimilar groups, and people interested in dynamical systems.”

According to Pilgrim, “I learned first of connections between complex dynamics and selfsimilar groups from a remarkable paper, ‘From Fractal Groups to Fractal Sets,’ by Rostislav Grigorchuk, Bartholdi, and Nekrashevych. I found this in 2002. There were so many new ideas! I spent the next several years trying to learn this stuff.”

In 2005, he met one of the paper’s authors, and they decided to organize a workshop at AIM on selfsimilar groups and complex dynamics. Pilgrim credits that workshop with bringing together these two communities, and introducing ideas that are now standard in complex dynamics.

In the following years, conferences in Switzerland and Denmark further developed these new



*Sarah Koch explains her fundamental diagram to Walter Parry (left) and Arturo Saenz Maldonado (right).*

perspectives. Pilgrim wrote a paper with Bill Floyd of Virginia Tech and Walter Parry of Eastern Michigan University on some aspects of the theory, and made partial progress on a number of other fronts. Meanwhile, Sarah Koch of Harvard University linked the topic with dynamics in several complex variables, and introduced new invariants and helpful non-dynamical points of view.

These mathematicians, as well as some of their students and former students, formed a SQuaRE that met at AIM for the first time this July to discuss algebraic approaches to Thurston maps. Pilgrim was excited about a new software program they were using: “You type in algebra, and you get out fractals,” he said.

“The SQuaRE model gives you the time needed to learn by getting your hands dirty,” said Pilgrim. He said this is especially important for younger mathematicians, who have to choose what to work on and who to work with. “Here at AIM, they have enough time and the promise of real outcomes, that it’s worth it to spend the energy and learn this new stuff. They learn how to marry their expertise. One mathematician may be an expert in algebra, but doesn’t have the geometric insight that someone else has, and doesn’t know his software. But now they can learn from each other.” ■

– Hana Silverstein

### Proposals require:

- a list of organizers
- a list of potential participants
- a description of goals
- an outline of how goals will be met

For more details and online applications:  
<http://www.aimath.org/research>

# Supporting Research Mentors

## Research Experiences for Undergraduate Faculty

AIM's Research Experiences for Undergraduate Faculty (REUF) program continues to thrive. Since the program began in 2008, approximately 100 faculty at primarily undergraduate institutions have participated in REUF workshops, spending a week engaged in mathematical research with colleagues and learning how to mentor undergraduate students on research projects. Because these faculty teach at institutions that do not have doctoral programs, they often devote most of their time to teaching and can have limited opportunities to pursue research themselves or with students. At REUF, undergraduate faculty work together with experienced faculty mentors on research projects that they can later involve their undergraduate students in. REUF also provides an opportunity for faculty participants to meet colleagues who are at similar institutions and to form new research collaborations themselves.

The outcomes of the REUF program so far demonstrate its effectiveness in encouraging faculty engagement in research with undergraduates, as well as with other faculty. Currently, more than half of REUF alumni begin mentoring undergraduates in research for the first time within one year of attending a REUF workshop. Many of these undergraduates present their work at conferences, with several having won grants or published papers related to their research. Some REUF faculty participants also form their own research collaborations, resulting in additional conference presentations and published papers.

Like last year's workshop, this year's was organized by AIM and hosted by the Institute for Computational and Experimental Research in Mathematics (ICERM) in Providence, Rhode Island. Because demand for the REUF program is high, with approximately 50 applications each year for 20 spots, it is hoped that the partnership between AIM and ICERM may lead to two workshops per year, one held at each institute.

This year's workshop was the first of three that will be supported by a National Science Foundation grant awarded to AIM, with Co-Principal Investigators Leslie Hogben (AIM and Iowa State University), Roselyn Williams (Florida A&M University), and Ulrica Wilson (Morehouse College and ICERM). The NSF grant also supports continuation activities so that faculty who begin working together at the REUF workshop have the chance to continue collaborating on their own research or on developing projects for undergraduate students. REUF continuation activities include week-long small-group research meetings held at AIM or another location, as well as informal lunches at national meetings such as MathFest and the Joint Mathematics Meetings. AIM also maintains an online discussion group and a resource website for participants.

For more information about REUF, please visit <http://www.aimath.org/reuf/>, or contact Leslie Hogben ([hogben@aimath.org](mailto:hogben@aimath.org)) or Brianna Donaldson ([brianna@aimath.org](mailto:brianna@aimath.org)). ■

– Brianna Donaldson and Leslie Hogben



Participants and facilitators at the 2013 REUF workshop pose for a group picture at the ICERM facility in Providence, Rhode Island.

# 100,000 Students and Counting

## Math Teachers' Circle Network Continues to Grow

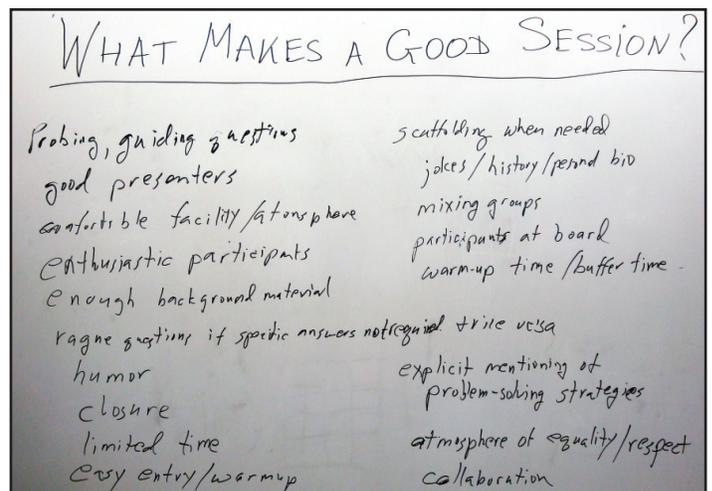
Math Teachers' Circles (MTCs) are spreading throughout the country, with nearly 60 active groups in over 30 states and a dozen more in the planning stages. Almost all of these groups have developed out of AIM's annual "How to Run a Math Teachers' Circle" workshops, which began in 2007. Each MTC is a community of middle school teachers and mathematicians who meet regularly to work on rich mathematical problems. With each MTC reaching 15 to 20 teachers annually, MTC participants are now reaching approximately 100,000 students across the country.

In the past year, generous gifts to AIM from the desJardins/Blachman Fund and Math for America have supported seed grants to new MTCs as well as continuing grants to existing groups. "These gifts have been crucial to ensuring the high success rate of new MTCs as well as the continuation of established groups," said AIM's Executive Director, Brian Conrey. "Groups have been very successful at using these funds to leverage additional local support." AIM also offers logistical support to the existing MTCs through the MTC Network (<http://www.mathteacherscircle.org>), which provides discussion groups, notes from successful math sessions, and a semi-annual newsletter.

According to the Conference Board of the Mathematical Sciences, "A substantial benefit of [Math Teachers' Circles] is that they address the isolation of both teachers and practicing mathematicians: they establish communities of mathematical practice in which teachers and mathematicians can learn about each others' profession, culture, and work." Ongoing research on MTCs, supported by a Discovery Research K-12 grant from NSF to AIM, is confirming the importance of this community to the teachers who participate. Preliminary results paint a picture of MTC teachers as highly professionally engaged and interested in translating their new mathematical experiences into their classrooms.

AIM is always looking for teams who are interested in beginning their own local MTC. Please contact Director of Special Projects Brianna Donaldson at [brianna@aimath.org](mailto:brianna@aimath.org) for more information. ■

– Brianna Donaldson



From top: Participants of the Heartland MTC in Olathe, Kansas, prepare to take a tree ring sample as a part of their summer 2013 immersion workshop. A board of ideas from the latest AIM workshop. MTC participants explore the geometry of the card game SET.

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# Meet the AIM Scientific Board

## Hee Oh: “Mathematics is Beautiful”



A few years ago, when Hee Oh's son was in the fourth grade, he decisively told his mother that it is a good thing she's a mathematician and not an elementary school math teacher. "I can't multiply numbers or add fractions correctly," she admits. And, in fact, it took her a long time to discover her penchant for mathematics.

"Until high school, I never thought I liked math," Oh said. "I had trouble understanding why double negatives become positive or why the sum of two sides of a triangle has to be greater than the third side."

As a grade school student, the South Korean native was convinced that mathematics was, in fact, quite contradictory to the real world. But one day in high school, she came home from school with a debilitating headache. Her sister later found her working on math problems. "Didn't you say you had a headache?" her sister asked. "Yes," Oh responded, "but I forget the headache when I am working on math problems."

When stiff competition kept Oh out of medical school after graduation, she took her elder brother's advice to study math instead, since there were many diverse career possibilities with a degree in math.

"At that time, I didn't even know there was a profession where one could make a living out of solving math problems," Oh said. "In retrospect, I thank God that I didn't go to medical school. I cannot stand seeing blood. And I love being a mathematician."

Oh tells that, in her first Calculus class, the professor wrote on the board, "Mathematics is beautiful."

"I was a bit shocked to see that," Oh said. "But it made me even more curious about mathematics. How could it be beautiful? I had to find out. Gradually, I found that I could see its beauty, and I could even find it for myself. It was fascinating! I still find great joy in understanding something I didn't understand before."

Oh went on to earn her Ph.D. from Yale University and to work and collaborate in Germany, the U.K., Israel, and various institutions in the U.S. She

has worked for the past several years as a professor at Brown University. Earlier this year, she took a new position as the first tenured female professor in the Yale University mathematics department. She is also a devoted wife to her husband, Seong-June Kim, and mother to 12-year-old Yoon-Young and 6-year-old Joy, which, at times, has been a source of some interesting challenges as she learns to find a balance between mathematics and motherhood.

She recalls one time when, asking a senior female professor for advice, the professor told her that she herself had quite prudently waited to get tenure before getting pregnant with her first child.

"Since I wasn't tenured at that time and was already pregnant with my first child, I could only look down at my big tummy and sigh, 'oops,'" Oh said. "But, in retrospect, I am glad that I didn't wait too long for my first child. Children give us so much joy and happiness."

Oh went on to add that mathematics actually has certain advantages as a career for a mother, because mathematicians tend to have more flexible schedules than many other professional careers. She has also learned how to use time more effectively and, thanks to some helpful advice from Peter Sarnak of Princeton University, has found a key element in the search for balance between mathematics and motherhood.

"My four years at Princeton were crucial for my career," Oh said. "Peter Sarnak had a huge influence in my mathematical interests, and exposed me to automorphic forms and analytic number theory. But he also gave me a key piece of life advice that I want to share with other young mother mathematicians."

"Buy as much help as you can with money," Sarnak had told Oh right after her first child was born, before promptly helping her hire a weekly cleaning person.

"You spend money on help with household chores, which frees up time to work on math," Oh explains. "And if you prove good theorems, you get promoted and your salary increases. So it is a great investment."

As a member of the AIM Scientific Board since 2010, Oh has thoroughly enjoyed having the chance to keep up with mathematical happenings in fields outside her own.

# Alan Reid: Leading A Double Life, Two Floors Apart



Every morning, Alan Reid takes the elevator to his office in the math department on the tenth floor of Robert Lee Moore Hall at the University of Texas at Austin, where his hours as a research mathematician are filled with hyperbolic manifolds, discrete groups, and low-dimensional topology. But

then, at noon, Reid goes down two floors to the administrative offices of the math department, where he serves as the department chairman.

“It’s a bit of a jarring gear-switch each day,” Reid said in a lilting Scottish accent. “Historically, the chairman is not research-active, and it requires discipline to keep the two spheres separate.”

Although Reid is honored to serve as the department’s chair, he said there is one major downside to wearing two different hats in the department.

“Sometimes the best mathematical problem solving happens in the mind’s background,” Reid said. “One might, for instance, have an epiphany while falling asleep. I find that since I’ve been chairman, this background noise is full of administrative burdens rather than mathematical problems. These days, my epiphanies are more along the lines of how to deal with budget constraints than mathematical breakthroughs.”

Reid had a special interest in numbers from a very young age, when he used to play counting games with his mother in the small fishing town of Buckie in northeastern Scotland, not far from where he went on to earn his mathematics degrees at the University of Aberdeen. He then went on to work and collaborate in places all over the world, including France, Brazil and New Zealand, before becoming a faculty member at the University of Texas at Austin.

“I am so fortunate that my job as a mathematician just allows me to come into my office and think about the things I like to think about,” Reid said. “I get to travel and go to interesting places and meet lots of interesting people.”

Of all the interesting people he has met, Reid considers himself particularly fortunate to have worked with Walter Neumann as a postdoc at the Ohio State University and to have a long-standing collaboration with Darren Long of the University of California, Santa Barbara, a collaboration that has resulted in 38 papers together to date. He also thoroughly enjoys mentoring and advising graduate students.

Reid has served on the Scientific Board of AIM since 2010, and he loves to read proposals and meet mathematicians from other areas of math that he might not otherwise come into contact with.

When Reid is not juggling his two mathematics jobs, he enjoys closely following international soccer leagues (“Well, I definitely call it football, though,” he said). He also enjoys playing tennis with his wife, Mara, a Brazilian he met in Austin. Reid has two sons, 18-year-old Mark, who is headed to college in Edinburgh in the fall, and 6-year-old Lucas, who enjoys traveling the world with Reid for his work and keeps his father busy with many questions about space and rockets. ■

– Jessa Barniol



Alan Reid steps into his role as mathematician with a series of three lectures at the 10th KAIST Geometric Topology Fair in Daejeon, South Korea, in August 2012.

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# Friends of the AIM Library

## A Special Thanks to Our Generous Supporters

Many university and public libraries have formal, structured friends groups whose members support their activities, programs and initiatives. Here at AIM, we have an informal, ever-expanding network of supporters who champion our mission to acquire and preserve books, offprints, journals and archives. Our “friends of the library” include research colleagues, workshop participants, mathematicians met at conferences, readers of the annual newsletter, and many others. Individually, they have secured numerous donations, both small and large, and strengthened the quality of AIM’s collections. While we still receive the majority of our donated items directly from the original collector or owner, we absolutely rely on the thoughtful efforts of our AIM boosters who spread the word and connect us with potential donors.

The most significant acquisitions of 2012-2013 were initiated by third parties. As always, Keith Dennis of the AIM Advisory Board alerted us to journal deaccessions. While we don’t actively solicit print journals, we certainly consider full, unbroken runs of significant titles. Thanks to Keith, this spring we acquired long runs of periodicals offered by IBM Research – Almaden, notably the *Bulletin of Symbolic Logic*, *Communications on Pure and Applied Mathematics*, *Zeitschrift für angewandte Mathematik und Physik* and several SIAM journals. And, from Bristol University, long-time AIM colleague Francesco Mezzadri put us in touch with Prof. Tim Browning, who coordinated Bristol’s donation of the first hundred years (1869-1969) of *Mathematische Annalen*. We are delighted to own such a key mathematical periodical.

Another friend of AIM is Samuel J. Patterson. On viewing the multiple reprint binders that ring the AIM conference room during a visit last August, he not only promptly donated his own sizable library of reprints, but spent a great deal of time and effort in organizing, listing and shipping the book collection of his Göttingen colleague, physicist Manfred R. Schroeder (1926-2009). Similarly, Peter Olver, who



*Pushpa Menon, Reprint Librarian*

recently arranged the donation of Walter Littman’s reprints, stepped in again to orchestrate Mrs. James B. Serrin’s donation of her husband’s archives. The contents of Prof. Serrin’s office at the University of Minnesota, comprising reprints, professional correspondence, lectures, notes, and manuscripts have been arriving in a steady stream of boxes since March and are currently being sorted and catalogued by Reprint Librarian Pushpa Menon (pictured above at work in the library) and her crew of library assistants.

The unique character of AIM’s library has been shaped by many generous donations, both directly and through its friends. Consequently, we have a collection that reflects the varied interests and activities of the broader mathematical community. Thank you to all!

A word about the back cover: Mathematical donations that arrive at AIM generally come at the close of a professional life of research, study, publication and achievement. We thought it might be appropriate to illustrate this edition of AIMatters with images of the doctoral dissertations that launched the careers of several eminent mathematicians. ■

*– Ellen Heffelfinger*

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# Prime Partners Exist

## Progress Toward the Twin Prime Problem

**Editor's note:** Kent Morrison and Brian Conrey created this abridged version of a longer article written by David Farmer. The full article is available on the AIM website at <http://aimath.org/news/primegaps>.

Number theorists view the integers as a largely unexplored landscape populated by interesting and friendly inhabitants. The prime numbers: 2, 3, 5, 7, 11, 13,... occur like irregular mileposts stretching off to infinity. The squares: 1, 4, 9, 16, 25,..., and the cubes: 1, 8, 27, 64, 125,..., along with the higher powers, are more sparse, but they also demand attention because of their special form.

These intriguing features exist because of the interaction between addition and multiplication. If we knew only about addition, then the integers would be quite boring: start with 0, then repeatedly add or subtract 1 to get everything else. End of story.

Things become interesting when you add multiplication to the mix. To construct every integer from addition, all you need is the number 1, but the building blocks for multiplication are the prime numbers, of which there are an infinite number. A great number of interesting questions—including some of the most famous unsolved problems of mathematics—deal with prime numbers. One of these questions is about the gap between consecutive primes. The smallest gap is between 2 and 3, but beyond that all primes are odd and so the gap between any two of them must be at least 2. These pairs of primes with a gap of two are called “twin primes,” and the great unsolved question concerning them is whether or not there are infinitely many of them. The question can be stated in a more mathematical way like this: Are there an infinite number of primes  $p$  such that  $p+2$  is also prime?

There are other interesting questions that can be asked in a similar way, such as: are there an infinite number of integers  $n$  such that  $n^2+1$  is prime? Are there an infinite number of primes  $p$  such that  $2^p - 1$  is prime? (These are Mersenne primes.) Are there an infinite number of primes  $p$  such that  $2p+1$  is prime? (Sophie Germain primes.) The simplicity of their statements belies the difficulty of their solution. None of

those questions has been answered, despite thousands of research papers and centuries of effort. But recently, Yitang Zhang of the University of New Hampshire has succeeded in proving a remarkable result related to the Twin Prime Problem.

You might just as well ask about primes  $p$  such that  $p+4$  is prime. Are there infinitely many of these? Or replace 4 by 6 or 8 or 10 or by any even number  $2n$ , and ask the same. The “prime neighbors” problem is this: Is there some even number  $2n$  such that there are an infinite number of primes  $p$  such that  $p+2n$  is also prime? Zhang proved that the answer is yes!

These other prime neighbors,  $p$  and  $p+2n$ , can actually occur more frequently than the twin primes. For example, if  $p$  is a prime, then  $p+6$  is more likely to be prime than  $p+2$  or  $p+4$ . This may surprise you, but the reason is simple. Since  $p$  is prime, it is not a multiple of 3, and when you divide it by 3, the remainder is either 1 or 2, with each equally likely to occur if you choose a prime at random. But in order for  $p+2$  to also be prime, the remainder must be 2, which eliminates half the primes from being the smaller in a pair of twins. Thus, a prime gap of 6 is twice as likely as a prime gap of 2.

A similar but slightly more involved analysis shows that a prime gap of 30 is  $\frac{8}{3}$  times as likely as a prime gap of 2. These observations are not actually proved theorems; they remain conjectures, but the numerical evidence supports them, as well as other conjectures about the frequency that other expressions give primes—expressions such as  $n^2+1$ . Credit for much of this goes back to the famous pair of Hardy and Littlewood, and although precise predictions supported by data are nice, we would prefer actual proofs.

For twin primes, Viggo Brun used a method now called the “Brun sieve” to prove in 1915 that there are not too many twin primes. He did this by proving convergence of the infinite sum of the reciprocals of the twin primes. We have known since Euler that the sum  $\sum_p 1/p$  of the reciprocals of the prime numbers  $p$  diverges, which means that there are relatively a lot of primes. However, Brun's result means that there are far fewer twin primes than primes, and provides negative

<del>1</del>	②	③	<del>4</del>	⑤	<del>6</del>	⑦	<del>8</del>	<del>9</del>	10
⑪	<del>12</del>	⑬	<del>14</del>	<del>15</del>	<del>16</del>	⑰	<del>18</del>	⑲	<del>20</del>
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⑦①	<del>72</del>	⑦③	<del>74</del>	<del>75</del>	<del>76</del>	<del>77</del>	<del>78</del>	⑦⑨	<del>80</del>
<del>81</del>	<del>82</del>	⑧③	<del>84</del>	<del>85</del>	<del>86</del>	<del>87</del>	<del>88</del>	⑧⑨	<del>90</del>
<del>91</del>	<del>92</del>	<del>93</del>	<del>94</del>	<del>95</del>	<del>96</del>	⑨⑦	<del>98</del>	<del>99</del>	100

evidence for the twin prime conjecture. It even allows for the possibility that there are only finitely many twin primes.

The big development, which is one of the main ingredients in Zhang’s recent work, is due to Goldston, Pintz, and Yıldırım in 2005, known as “GPY.” It is GPY, combined with some techniques from the 1980’s, which made Zhang’s result possible. All the GPY papers are in the AIM preprint series, available at <http://aimath.org/preprints.html>. The GPY method was the topic of the November 2005, AIM workshop “Gaps Between Primes.” An interesting sidenote is that the workshop was the first occasion that Goldston, Pintz, and Yıldırım were all in the same place at the same time!

Key to the GPY method is knowledge about the distributions of primes in arithmetic progressions. An arithmetic progression is an infinite sequence of positive integers equally spaced apart, such as the sequence beginning 1, 4, 7, 10, 13,...., in which the spacing is 3. There are two other arithmetic progressions with the same spacing of 3, namely 2, 5, 8, 11, 14,... and 3, 6, 9, 12, 15,.... Obviously, there is only one prime in the last sequence, but there are many primes in the other two. According to a famous theorem of Dirichlet (1837), if an arithmetic progression has no common factor, then it will contain infinitely many primes.

In 1899, de la Vallée-Poussin went further, showing that the primes distribute uniformly—in a probabilistic sense—among the progressions that contain primes. Thus, a prime is equally likely to be found in either of the two progressions 1, 4, 7, 10, 13,... and 2, 5, 8, 11,

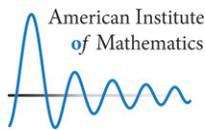
14,.... Consider the case in which the spacing is equal to 15. Now there are eight different progressions that contain primes: those starting with the numbers 1, 2, 4, 7, 8, 11, 13, and 14. In the long run, the prime numbers spread themselves out evenly among the eight different progressions.

Zhang modified the GPY method by restricting attention to numbers having no large prime factors—known as “smooth” numbers. Concentrating on smooth numbers gave Zhang the flexibility that he needed in order to use techniques developed by Bombieri, Friedlander, and Iwaniec (known as “BFI”), who, in a series of three papers in the late 1980’s, figured out how to do a better job estimating the average behavior of primes in arithmetic progressions.

The BFI results were well known to the experts, and combining the GPY and BFI methods to obtain bounded gaps was an obvious next step in the eight years since the GPY result, but Zhang had the insight to modify the BFI techniques to make them work.

Zhang’s original paper shows that the number  $2n$  exists, which is the breakthrough result that surprised everyone, but he did not determine it exactly. He does show that  $2n$  is less than 70,000,000, a number which he made no effort to optimize, but which has since been improved significantly by others refining Zhang’s approach and methods. As of this writing, the best confirmed improvement is a prime gap of at most 5,414, and a not-yet-confirmed improvement to 4,680. At the rate things are going, there will probably be a better result by the time you read this! ■

– David Farmer

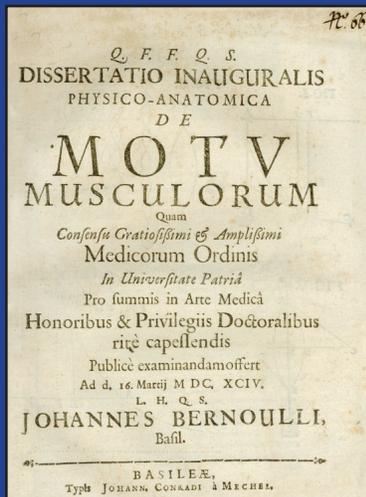


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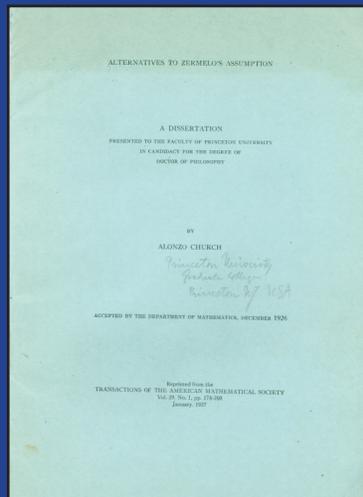
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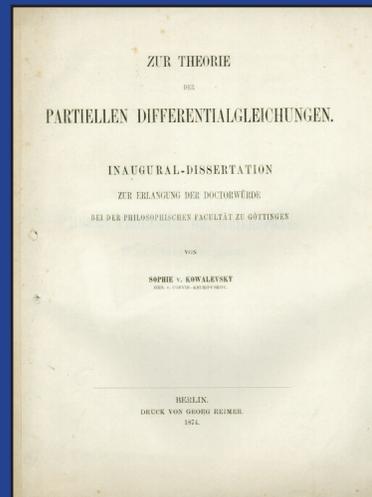
## A Selection of Dissertations



Bernoulli, 1694



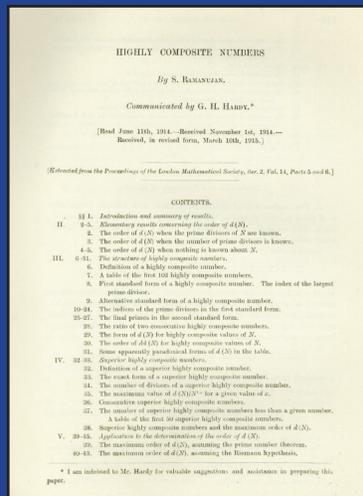
Church, 1927



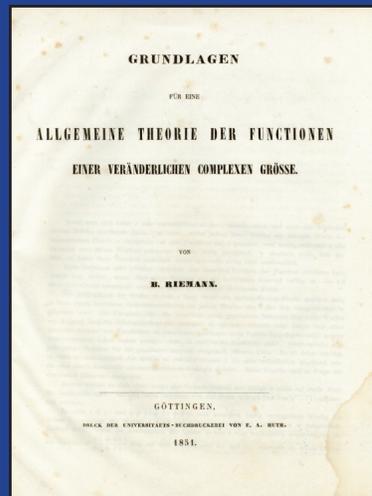
Kovalevskaya, 1874



Nash, 1951



Ramanujan, 1915



Riemann, 1851