Autumn 2014

AIMatters

Newsletter of the American Institute of Mathematics

CELEBRATING 20 YEARS OF AIM

From Kadison-Singer to Berries + **Math Research and the Deaf Community** + **Textbooks: You Get What You Pay For?**

Celebrating 20 Years of AIM

Letter from the Director

The first meeting of the Board of Trustees of the American Institute of Mathematics was convened by Board Chairman Gerald Alexanderson on June 28, 1994. John Fry's dream of creating an institute that fostered collaborative efforts to solve important questions in mathematics had been realized. Now, twenty years later, we look back at some of the exciting things that have happened in the life of AIM.

An early success was the solution of the Perfect Graph conjecture. In the fall of 1998, Paul Seymour, Robin Thomas, and Neil Robertson began working on the problem full time, supported by AIM. Four years later, with the addition of Maria Chudnovsky to their team, they succeeded!

Around that time AIM received funding from the National Science Foundation as a Mathematical Sciences Research Institute with the mission of hosting focused collaborative workshops. One of the first such workshops was about the Perfect Graph Conjecture and it was during this workshop that the Perfect Graph recognition problem was solved! Now, 236 workshops later, AIM can lay claim to having begun a new style of workshop that emphasizes planning, discussing, and working in groups to advance a specific subject. In a number of cases, the problem lists created by workshop groups have helped set an agenda that steers a field for years.

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Letter from the Director

When AIM's institute grant was renewed in 2007, the Structured Quartet Research Ensembles (SQuaREs) program was initiated. This program was originally intended to support the small groups that formed during workshops. Now it has a broader reach and is open to any small group (4 to 6 people) with an ambitious project that has the potential for significant progress over the course of three week-long meetings separated by a year. AIM has sponsored 136 SQuaRE weeks; it has proven to be a very popular program!

In 2006, AIM started the Math Teachers' Circle (MTC) program. MTCs are communities of middle school teachers and mathematicians who meet regularly to solve interesting math problems. There are about 70 MTCs all over the country now. Originally designed "to get middle school teachers to think about math the way math researchers do," we now see the MTC program as a vital component in the professional development of teachers and as a model consistent with the Common Core State Standards, especially the "practice standards" which are about problem solving. Our MTCs now reach about 1 percent of all middle school teachers in the U.S. Our eventual goal is to have an MTC within one hour's drive of every middle school teacher in the country. Mathematics departments could play a significant role in such an effort. If every math department sponsored an MTC, we would move a long ways towards this goal, AND it would be a great way for math departments to stay in touch with the teachers they have taught, those who are on the front lines of our community's effort to make our country more numerate.

One of the very first initiatives of AIM was its Five-Year Fellowship. The idea was to choose one new mathematics Ph.D. each year and fully support them for the first five years of their research career. AIM has appointed 13 Five-Year Fellows, the first being Kannan Soundararajan in 1998 and the last being Melanie Wood and Kirsten Wickelgren in 2009. These 13 have distinguished themselves in numerous ways and have contributed many of the 796 papers on AIM's preprint page. We are very proud that Jacob Lurie of Harvard University, who was named the AIM Five-Year Fellow in 2004, is one of the first recipients of the prestigious Breakthrough Prize in mathematics. This award carries with it a $3 million honorarium! While an AIM Fellow, Jacob wrote a 925-page treatise, "Higher Topos Theory," which has been published in the Annals of Mathematics Studies by Princeton University Press. The Math Review reads in part, "This book is a remarkable achievement, and the reviewer thinks it marks the beginning of a major change in algebraic topology." Well done, Jacob!

We look forward to more exciting advances in the next 20 years! ■

– Brian Conrey

A Flourishing Partnership
AIM and Julia Robinson Mathematics Festivals

The 2013-2014 academic year was AIM's first in partnering with the Julia Robinson Mathematics Festival, which allows students who thrive on collaborative problem solving to have fun working with skilled mathematicians and one another. Launched in 2007 to provide an alternative to the traditionally competitive culture of math contests, the Festival offers a range of intriguing, challenging problems, puzzles, and activities.

This year we were able to significantly expand the number of events around the country, including new Festivals in the Navajo Nation and Columbus, Ohio, and at Princeton University and the San Mateo County Office of Education. Also, after a hiatus of a few years, the Festival returned to the campuses of Google and Pixar Animation Studios. Demand continues to grow, and we hope to greatly increase the number of Festivals in the 2014-2015 academic year. If you are interested in hosting, sponsoring, or co-sponsoring a Festival in the coming year, please contact us at **info@juliarobinsonmathfestival.org**. ■

– Mary Eisenhart

Students work together on an untying problem at this year's Julia Robinson Math Festival at UC Berkeley.

Thank You From AIM
In Gratitude To Our Generous Library Donors

We gratefully acknowledge the following donors for their generous contributions of books, reprints, journals and archives to the AIM Library in 2013 and 2014.

Gerald L. Alexanderson
American Mathematical Society
Michael Artin
Bryan J. Birch
Family of Nelson M. Blachman
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Robert B. Burckel
Francis J. Flaherty
Leonard Gross
J. William Helton
Family of Thomas Robbins
Richard D. Schafer
Alan D. Weinstein
Hung-Hsi Wu

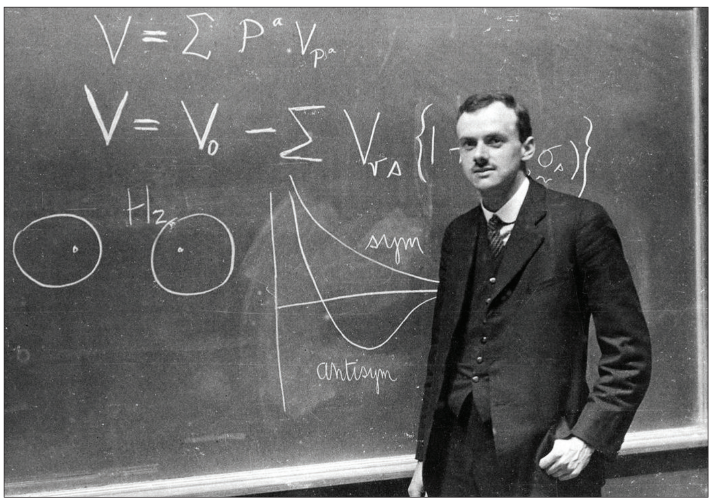
– Ellen Heffelfinger

From Kadison-Singer to Berries

AIM Facilitates a Wide Range of Research Projects

One of the remarkable things about mathematical research is how vast and encompassing it is. This is most apparent at AIM as new workshop and SQuaRE participants arrive weekly and describe their projects. During the past year, two problems at different ends of the research spectrum, but both with close AIM connections, received national media attention.

The first of these was the resolution of the Kadison-Singer problem, solved by Adam Marcus, Dan Spielman, and Nikhil Srivastava. The problem appeared in different guises in mathematical physics, operator theory, complex analysis, graph theory, frame theory, signal processing, and, ultimately, as a problem in finite-dimensional geometry. In each of those areas the problem has a precise formulation, but, interestingly, some of the versions were believable, while others seemed quite unbelievable.



From top: A historical photo shows Paul Dirac at work. Kathleen Fowler of the AIM berry project presents AIM Director Brian Conrey with a set of cookies she baked as a gift for AIM. The cookies include numbers, a pi symbol, the AIM logo and the door mural.

The first formulation goes back to Paul Dirac, who asked whether the measurements of one laboratory observable determine uniquely the observables that, by the uncertainty principle, cannot be measured at the same time. In 1958, the problem was translated into the language of operator theory by Richard Kadison and Isadore Singer where it became a statement about whether or not a "state" (a certain type of functional) defined on an abelian subalgebra of operators extends uniquely to the set of all bounded self-adjoint operators.

In 1980, a new formulation called the "paving conjecture" was discovered by Joel Anderson. In the meantime, a number of different conjectures in different fields were put forward before Pete Casazza, along with three other mathematicians, realized in 2006 that all the conjectures were equivalent. In the same year, Casazza, Kadison, and David Larson organized the "Kadison-Singer Problem" workshop at AIM.

Out of the workshop came one of the first AIM SQuaREs consisting of Casazza, Bernhard Bodmann, Vern Paulsen, and Darrin Speegle. Their goal was to resolve the original conjecture or one of its equivalences, and while that was not successful, they produced a counter-example to a generalization of a 20-year-old result of Bourgain and Tzafriri, which was believed to be true, resolved a 28-year-old version of a paving conjecture, and proved important results in frame theory. So both the workshop and the SQuaRE provided a way to keep the Kadison-Singer problem on the radar of mathematicians for long enough to become a problem that would capture the interest of others.

In the end, Marcus, Spielman, and Srivastava proved another equivalent version, the Weaver conjecture, which is closely related to the Anderson paving conjecture. They showed that the characteristic polynomials that are relevant to the Weaver conjecture form an interlacing family of polynomials.

Coincidentally, it turns out that a 2007 AIM workshop, "Polya-Schur-Lax problems: Hyperbolicity and stability preservers," studied related problems. Elements of the work done by the late Julius Borcea and Peter Branden following the workshop can be seen in the solution of the Weaver conjecture.

In December 2014, a followup workshop called "Beyond Kadison-Singer: Paving and consequences" organized by Casazza, Marcus, and Spielman, will focus on the applications of the conjecture.

The other AIM activity that has been in the news is a "berry project," an attempt to help berry growers in the Pajaro Valley of California balance water usage and sustain crop volume. The National Science Foundation featured the research in a video that appeared in their Science Nation series: "Strawberry fields forever — with some help from mathematicians!" Both the video and a written version can be seen at **http://www.nsf.gov/news/overviews/mathematics/**.

This project started with the 2011 Sustainability Problems workshop, which brought together mathematicians and industry representatives to work on a variety of sustainability problems, including renewable energy, air quality, water management, and other environmental issues. One of the industrial participants was Driscoll's, whose associated growers are the largest supplier of fresh berries in North America. Three Driscoll's employees teamed up with nine applied mathematicians to evaluate how various water and land management techniques could be utilized by landowners and growers to work towards balancing aquifer levels. After the workshop, a SQuaRE group led by Lea Jenkins and Katie Fowler made significant progress in the creation of a virtual farm model to study alternative crop management strategies and their effect on water usage and profit.

The current version of the virtual farm model implements a framework for simulating a 100-acre model farm. The framework acts as a virtual farmer, planting and moving crops around the farm using various planting rules as constraints. The profits realized from the virtual farm and irrigation constraints are computed. These values are used to evaluate strategies using different objectives such as maximizing profit, minimizing water usage, and minimizing deviation from current demand.

The team also investigated a surface water analysis to understand feasible ways to capture rainfall for re-infiltration (or recharging) into the aquifer. The latest version of the farm model also includes scenarios under which stakeholders could allocate land for the construction of a managed aquifer recharge network, giving optimal sizes of the recharge basins based on maximizing recharge while minimizing construction cost. ■

– Estelle Basor

From top: Berries growing in a California field. The berry project participants, from left to right, Matthew Farthing, Lea Jenkins, Kathleen Fowler, Stacy Howington, John Chrispell, and Corey Ostrove, visit a raspberry field at JE Farms in Watsonville, California.

Raspberry photo: Nathan Ford, stock.xchng.

Signs of the Times

Bringing Research Experiences to Deaf Students

Bonnie Jacob is an assistant professor at the National Technical Institute for the Deaf (NTID), which is one of the nine colleges that comprise Rochester Institute of Technology (RIT). There, she works with deaf and hard-of-hearing students who are enrolled in math classes across the campuses with hearing peers and hearing professors.

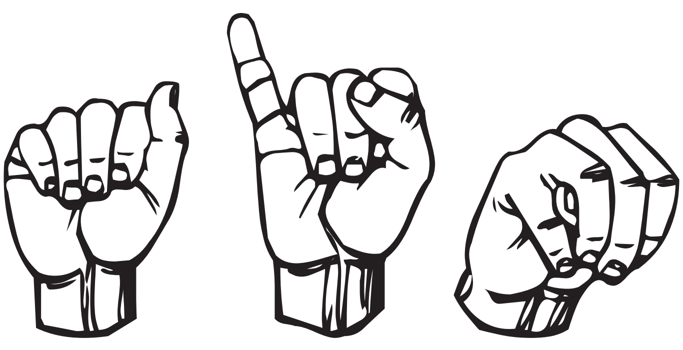
"Before I came here, I knew nothing about Deaf culture," said Jacob. "I was overseas in Japan, where they teach Japanese Sign Language, and I happened to meet deaf people and became interested in being involved in the Deaf culture."

There is a natural intersection between Deaf culture and the mathematics world. A lot of mathematics is about seeing patterns and thinking about challenging problems with minimal use of language, and this appeals to deaf students. "When it's a problem that a student can grasp without struggling through a lot of English, they really grab onto that," said Jacob.

Yet deaf and hard-of-hearing students often lag behind their hearing peers in mathematics. The multiple challenges of communicating with teachers who do not sign and jumping into a situation that is mathematically unfamiliar can be formidable.

In her second year at RIT, Jacob started mentoring a deaf undergraduate student on a project on wavelets in financial mathematics. "He was delighted," Jacob said.

The experience showed her that there were deaf students who were interested in research mathematics and had the background to pursue it. If she could work with more deaf students on a wider variety of projects, perhaps many of them would gain the confidence to seek out research opportunities with hearing faculty and peers.



"AIM" as finger-spelled in American Sign Language.

In 2012, Jacob attended an AIM-sponsored program called Research Experiences for Undergraduate Faculty at the Institute for Computational and Experimental Research in Mathematics (ICERM). The week-long program introduced Jacob to other young undergraduate faculty who were interested in mentoring undergraduate research. All week, participants collaborated on open-ended problems, engaging in research similar to what they hoped to do with students back at home.

Thanks to a Center for Undergraduate Research in Mathematics grant, funded by the National Science Foundation and by Brigham Young University, Jacob began looking for students to form a research group in 2013. She approached three undergraduates who were majoring in information technology and computer science – Thomas Ansill, Jaime Penzellna, and Daniel Saavedra. One was enrolled directly at RIT; the other two had started at NTID at the associate's level and were at different stages in moving on to the bachelor's level. All were mathematically curious, worked well together, and had taken discrete mathematics, which was to be the topic of the research project.

"I often went to Bonnie Jacob for tutoring on discrete mathematics," said Thomas Ansill. "She noticed my problem-solving skills and offered me a position in her research group."

"I had done a little work with students before, but informally, and with no pay and no credit," said Jacob. "I was in a daze at the time because I was worried about everything."

Jacob wanted to start with a problem that the undergraduates could work on right away. On day one, she posed a question and told them to try it on a very simple graph. She emphasized that the problem was new, and hadn't been done before. The students started playing around with it immediately.

The goal was simply to engage students in math research, to get them interested in exploring unknown problems. One of the toughest parts of the research project, according to Ansill, was "finding a simplified and minimal solution to most problems." Another challenge was developing an efficient computer program to solve problems.

Jacob noticed how the students often took a meandering path in their explorations. "One day, one of my students was working on something, and he started trying to relate two things to me – the number of vertices in this versus the number of vertices in that," she said. "I thought, why is he doing that? But it was new to him, and interesting to him."

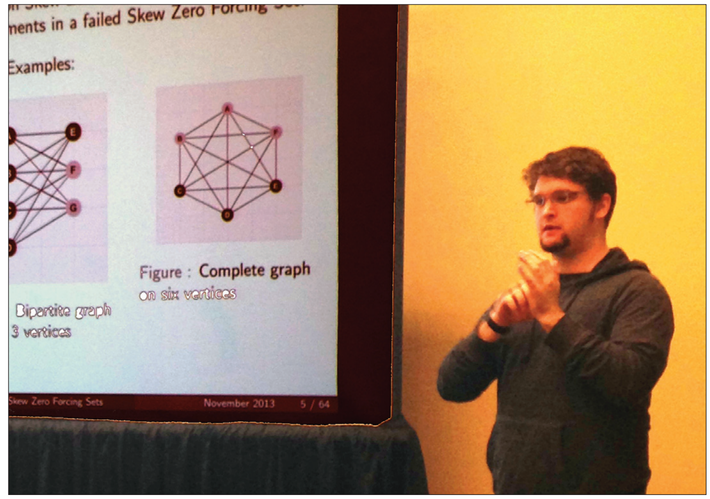
Students also explored new ways to communicate. They signed the letter "V" to mean vertices, and came up with appropriate signs for other words. RIT had a rich pool of academic sign language to draw on, but if there was no sign for a term, students would finger-spell it, or go back and forth between writing and signing.

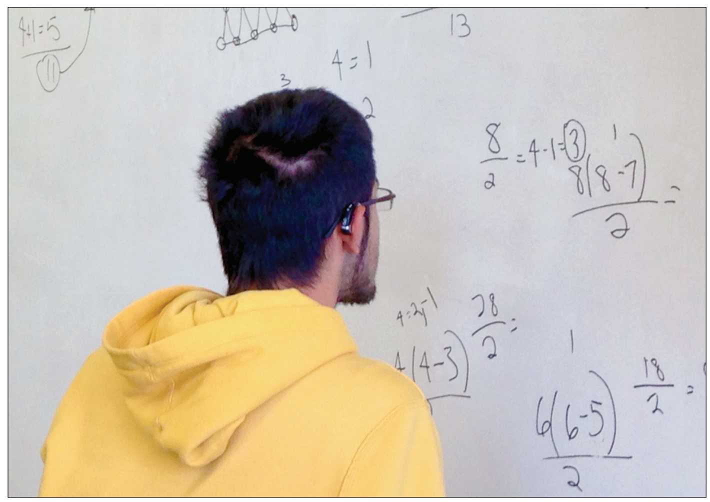
In January 2014, Jacob and the students traveled to Baltimore for the Joint Mathematics Meetings. The students gave a talk titled, "Failed zero forcing and failed skew zero forcing on graphs." It was their first professional mathematics conference. "One of the students just shocked me when someone, a non-mathematician, came up and asked about what we were doing," said Jacob. "The student is really good at explaining in a way that someone who doesn't know math can get what's going on."

Jacob hopes that the undergraduate research experience will have an impact beyond the scope of her small group. "Research is not part of American Deaf culture, so as students talk about it with friends, I hope it becomes a possibility for more deaf students," she said.

For the students, the impact was more immediate. "It was a great experience!" said Jaime Penzellna. "Collaborating with other students and learning from each other was the best tool there is. The biggest challenge was finding patterns to the skew zero forcing sets, but eventually, after enough experimenting, we found very interesting things!" ■

– *Hana Silverstein*





From top: Thomas Ansill presents at the Joint Mathematics Meetings in Baltimore. Daniel Saavedra works on formulas for the failed skew zero forcing numbers of different classes of graphs. Thomas Ansill and Jaime Penzellna tackle the same problem.

Paving the Way for Research

Training and Connecting Undergraduate Mentors

The Research Experiences for Undergraduate Faculty (REUF) program brings together faculty members from colleges and universities that emphasize undergraduate education for a one-week workshop where they work with experienced faculty leaders on research projects that are accessible to undergraduate researchers. At each REUF workshop, there are four leaders and 20 faculty participants. During the first morning, the leaders present problems, and for much of the rest of the week the participants are immersed in doing research in groups with the leaders and each other. Many REUF participants go on to mentor undergraduate students on research related to their REUF project.

REUF began at AIM in 2008 and is a partnership of AIM and the Institute for Computational and Experimental Research in Mathematics (ICERM). The REUF workshop now alternates between institutes and was held at AIM in 2014. Research groups formed at REUF workshops also have an opportunity to apply for a second week together a year after the workshop to continue their research and prepare results for publication. This second week, which can be held at AIM, ICERM, or another site, is called a REUF SQuaRE, since it is modeled in part on the AIM SQuaREs program for small groups of researchers. Some of these groups have lead to even longer-term collaboration: A 2012 REUF group led by AIM's Associate Director for Diversity Leslie Hogben that reconvened in 2013 at Iowa State University is making plans to hold a third meeting in summer 2015.

Other sustaining activities focus on building a community of REUF alumni to support undergraduate research and collaboration. Brianna Donaldson, AIM's Director of Special Projects, maintains a REUF alumni listserv and REUF website at http://reuf.aimath.org/ and organizes REUF alumni lunches each year at the Joint Mathematics Meetings in January and MAA Mathfest in August.

REUF serves many junior faculty, for whom research activity and supervision of undergraduate research students are becoming tenure expectations, as discussed by Adriana Salerno (Bates College, REUF 2011) in her blog post http://blogs.ams.org/phdplus/2011/08/25/raise-the-reuf. Tenured faculty face different challenges, as expressed by REUF 2012 participant Cheryl Grood: "With the heavy teaching and service demands at these institutions, it can be easy for one's research program to slow down post-tenure, which then can compromise one's ability to supervise student research." REUF can also provide new directions and collaborations that lead to research renewal for these faculty.

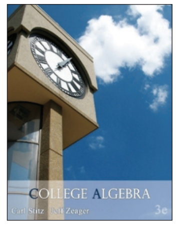
For more information about REUF, please visit http://reuf.aimath.org/, or contact Leslie Hogben at hogben@aimath.org or Brianna Donaldson at brianna@aimath.org. ■

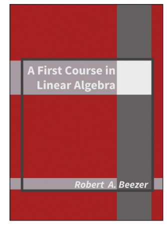
– Brianna Donaldson and Leslie Hogben

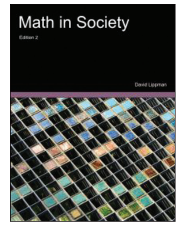
Left: Workshop participants discuss issues surrounding mentoring undergraduate research. Right: The "happy numbers" research group from REUF 2014, led by Helen Grundman of Bryn Mawr College.

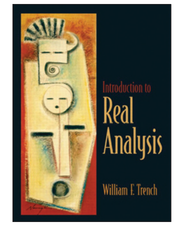
AIM Explores Open Textbooks

Do You Really Get What You Pay For?









Here are your choices for a linear algebra textbook. The first is the biggest seller in U.S. colleges and universities and is published by one of the giant publishers. From Amazon a new hardcover copy costs $141 and the Kindle edition is $91. The second is "A First Course in Linear Algebra" by Rob Beezer. From Amazon you can buy a copy for $35, but the PDF is completely free and can be read on your computer or tablet (or even a phone), and there is also an absolutely free HTML version that can be read anywhere you have an internet connection.

The choice is clear if your main concern is price, but what about quality? The first book is an excellent text and is recognized as such. The second is also an excellent book, but most faculty choosing textbooks do not believe that a free or almost free book can be as good as an expensive one, and so we set out to do something about it—at least for undergraduate mathematics texts. Beginning in 2010, the AIM Open Textbook Initiative has worked to identify open source and open access math books that we can recommend for serious consideration by faculty when they are choosing a book for course adoption. We have evaluated more than 50 books and found 30 that meet specific, well-established criteria. The work is carried out by an editorial board consisting of five math professors with decades of teaching experience representing institutions ranging from community colleges to research universities.

Our efforts are directed toward college faculty, and although they are not the ones actually buying the books, there are clear reasons they might prefer open access texts for their students. First and foremost, every student in the class will have a book. Sadly, it is a common problem for a sizable percentage of a class to not buy the assigned text because of financial hardship. Furthermore, with laptops and mobile devices, students can have their books in class and a printed copy at home. With the commercial text above the total cost is more than $230 for both versions. And another benefit is not having to deal with new editions every three or four years that are produced mainly to counteract the used book market.

The AIM Open Textbook Initiative can be found at **http://www.aimath.org/textbooks/**.

For more information, read the following two articles about the initiative, available without subscription: "What We Are Doing About the High Cost of Textbooks," Notices of the Amer. Math. Soc., August 2013, pp 927-928. (**http://www.ams.org/notices/201307/rnoti-p927.pdf**) and "The AIM Open Textbook Initiative," MAA Focus, Dec. 2013/Jan. 2014, p 35. (**http://digitaleditions.walsworthprintgroup.com/publication/?i=187509**) ■

– Kent Morrison

CALL FOR PROPOSALS

Proposals are currently being sought for week-long workshops for up to 28 people and SQuaRE collaborations for 4-6 researchers to take place in 2015-2016 at AIM in Palo Alto, California.

Proposals require:

- **a list of organizers**
- **a list of potential participants**
- **a description of goals**
- **an outline of how goals will be met**

http://www.aimath.org/research

Application deadline: November 1, 2014.

The Future Begins Now
Morgan Hill Math Nurtures Young Mathematicians

AIM Morgan Hill Math, now in its eleventh year, continues to evolve as it seeks to fulfill its mission of "nurturing young mathematicians."

"This is the funnest math day ever!" exclaimed a third-grade student attending AIM's Math Fest at El Toro Elementary School in Morgan Hill, California, as she worked to complete a pattern of unifix cubes. AIM Morgan Hill Math has once again succeeded in challenging a large number of students in the 2013-2014 school year, offering math programs at various levels for students in third through twelfth grades. This year, AIM expanded its offerings to include a third-grade Math Fest and a spring initiative for underrepresented minorities, sponsored by a Tensor SUMMA grant.

One of the guiding principles of AIM's Morgan Hill Math program is to make sure that each student in the program feels sufficiently challenged, never bored nor stifled. The entrance exam for fourth, fifth and sixth graders seeks to find students who love math. Comments on our exit survey show that we are succeeding, with more than one student responding to the question "What was your least favorite thing about Mathletics?" with the answer "That it's over." Elementary students who do well in AIM's Mathletics and MathCounts6 programs further challenge themselves by competing in the national Math Olympiad program from November to March.

Those students who exhibit insight and the patience required for deep thinking are invited to work directly with AIM's Executive Director Brian Conrey in our Director's Circle program.

The mainstay of AIM Morgan Hill Math continues to be a sixteen-week afterschool program for seventh and eighth graders preparing for the annual MATHCOUNTS competition. For the second year in a row, a group of four students from Silicon Valley Flex Academy, lead by AIM coach David Holmstrom, triumphed at the chapter competition and went on to the Northern California State MATHCOUNTS competition held at Stanford University on March 22, 2014.

High school students in Mu Alpha Theta (a math honors society) challenged themselves as they studied for the 2014 American Mathematics Competitions (the AMC10 and AMC12) in a self-formed study group. Mindful of teaching their younger counterparts, they ran a seminar at Science Alive on toothpick structures and another half-day event that they dubbed "The Amazing Math Race." Katie Grant, a long-time AIM Morgan Hill Math student, bridged the worlds of work and college by interning at AIM this summer. She heads to UC San Diego this fall, majoring in economics and mathematics. A future mathematician launched! ■

– Lori Mains

Left: AIM Director Brian Conrey discusses fractals with fifth and sixth graders during the Director's Circle meeting. Right: Elementary school teacher Debbie Stewart listens as one of her students explains why the three cards on the table form a valid "set" in the game of SET.

Creating Teaching Communities
Math Teachers' Circles Support Teachers

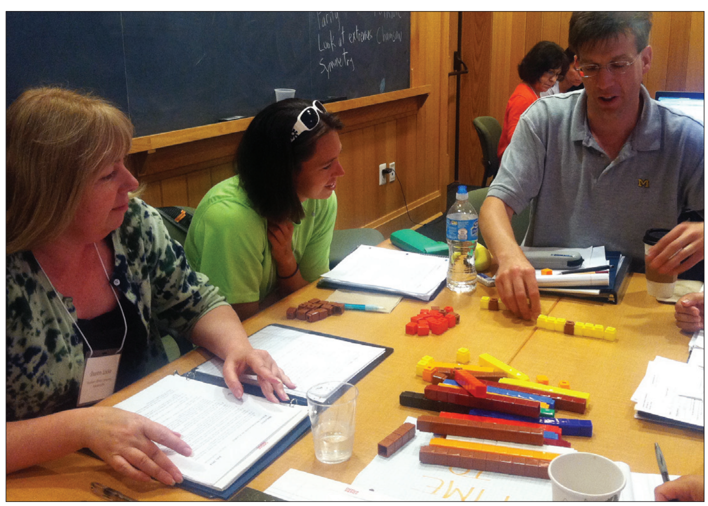
Math Teachers' Circles (MTCs) are professional development communities of middle school teachers and mathematicians who meet regularly to work on interesting mathematics together. There are approximately 75 active and planned MTCs across the United States, each of which reaches about 15-20 teachers per year. AIM has played an active role in starting and sustaining these groups, through efforts such as organizing annual "How to" workshops, publishing a semi-annual newsletter, and maintaining a website with materials and resources at **http://www.mathteacherscircle.org**.

In 2011, AIM was awarded a Discovery Research K-12 grant from the National Science Foundation to study how participating in MTCs affects teachers. As part of the grant, we conducted a national survey of over 200 teachers who participate in MTCs. Belonging to a professional mathematical community was the most commonly cited reason for participating in MTCs. Two-thirds of the survey respondents told us that they now have a better understanding of what is involved in doing mathematics. "My ability to persevere in solving problems has improved," said one teacher. Additionally, 84 percent of respondents identified ways that MTC participation has affected their teaching. Many reported bringing more interactive activities and non-routine problems to the classroom. Others said that they are giving students more time to think and collaborate on problems.

Finally, 44 percent of respondents wrote that MTCs had significantly affected their professional lives outside the classroom. "It has opened many doors for my future," wrote one teacher. A common observation among respondents was that MTCs provide scholarly and social support that isn't always found in other forms of professional development. "It is helpful to have an ongoing professional development community," said one teacher. "The opportunity to practice active research is helpful."

AIM is always looking for teams who are interested in beginning their own local MTC. Please contact Brianna Donaldson, Director of Special Projects, at **brianna@aimath.org** for more information. ■

– Brianna Donaldson and Hana Silverstein



From top: A team from Bowling Green State University creatively presents their MTC plans. Workshop participants enjoy a tour of the Mathematical Association of America's headquarters. Participants investigate how many distinct hexominoes can be created.

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Benefiting the American Institute of Mathematics

October 6-12, 2014

Math in the Afternoon

Lunchtime Bocce at AIM Spurs Paper on Probability

A game of bocce after lunch is a regular activity at AIM. The AIM staff often heads over to a nearby park to eat lunch outside and play a game. There is no bocce court in the park; this game is a free-form version on grass, in dirt, and even in the sand of the playground.

We play in teams or as individuals—depending on the number of players and how much time we have. The game ends when the first team or individual reaches 11 points. The quickest games are those with two teams and four balls per team, because anywhere from 1 to 4 points may be scored in each round.

In bocce, the players toss large colored balls toward a small white target ball, the pallino. The team with the colored ball closest to the pallino receives one point for that ball and additional points for each additional ball that is closer than all of the opponents' balls. The player with the winning ball then tosses the pallino out into the park somewhere in order to begin the next round. That leads to strategizing, since players have different strengths and weaknesses in their game. Some like the challenge of throwing between trees or throwing near a sidewalk. (According to AIM rules, any hard pavement is out of bounds, and players' balls that hit pavement are out of play and cannot score.)

Look at the two photos. See a difference? In the left photo the pallino is enclosed by the players' balls, but that is not the case on the right. If you were to tightly place a string around the colored balls, then the polygonal area enclosed is the "convex hull" of those balls. So, in the left photo, the pallino is within the convex hull of the colored balls, and in the right photo it is not.

A few years ago the arrangement as in the right hand photo struck us as unusual, and, being mathematicians, we asked, "Just how unusual is it?" In our bocce games, the numerous obstacles and uneven terrain lead to a large degree of randomness in the balls' trajectories, and so, to answer our question, we made some reasonable probabilistic assumptions. First, we assumed that the location of each of the players' balls is independent of the others. Second, we assumed that the direction from the pallino to a player's ball, which can be represented by a point on a circle with the pallino at the center, is uniformly distributed on the circle. With just these two assumptions you can calculate the probability that the pallino is outside the convex hull of the players' balls. For the most common game that we play, one with 8 balls, that theoretical probability is 1/16, which certainly qualifies as "unusual." For n balls, the probability is $n/(2^{(n-1)})$.

These speculations led me to write the article "From Bocce to Positivity: Some Probabilistic Linear Algebra," Mathematics Magazine 86 (2013) 110-119. The original bocce question turns out to be the same mathematically as several other questions, which are all examined in the article. Read more at **http://www.calpoly.edu/~kmorriso/Research/BocceMM.pdf**. ■

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From Our Collections

Three items from the personal library of **Carl Friedrich Gauss**, *including Gauss's logarithmic tables; a pamphlet on spherical trigonometry inscribed by the author, Erasmus Thune, to Gauss; and the dissertation of Richard Dedekind, Gauss's last student.*